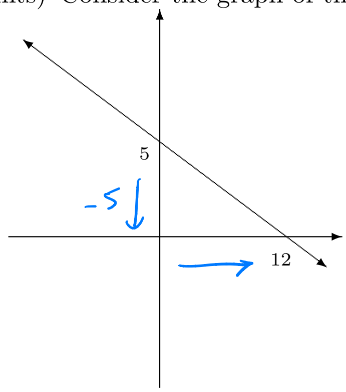
Updated May 13, 2024 3:15 pm

H/G HATT Ch 2 Practice Test Name: Block: Seat:

1. (4 points) Consider the graph of the line below:



   (a) What is the equation of the line in the graph? (use either general or slope intercept-form)

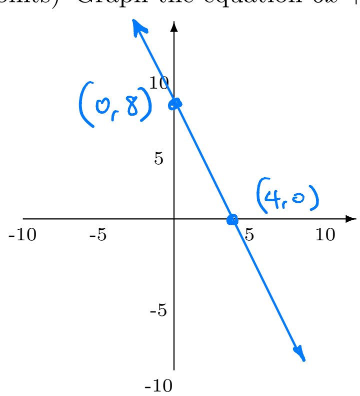
   $y = -\frac{5}{12}x + 5$

   $5x + 12y = 5$

   (b) What is the equation of the line perpendicular to this line that passes through the point $(12, 0)$?

   $y = \frac{12}{5}(x - 12)$

2. (2 points) Graph the equation $6x + 3y = 24$ below *(be sure to label two points on the line)*



If $y=0$ $x=4$ $(4,0)$

If $x=0$ $y=8$ $(0,8)$

3. (2 points) Find the exact distance between $(5, -3)$ and $(-1, -7)$.

$\sqrt{(5-(-1))^2 + (-3-(-7))^2}$

$\sqrt{6^2 + 4^2}$

$\sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

4. (2 points) What is the midpoint between $(5h, -3\pi)$ and $(7h, -7\pi)$?

$\left(\frac{5h+7h}{2}, \frac{-3\pi+-7\pi}{2}\right)$

$(6h, -5\pi)$

5. (2 points) List the intercepts for the graph of the equation $4x^2 + 16y^2 = 64$.

If $x = 0$,

$y^2 = \frac{64}{16}$ $y = \pm 2$

If $y = 0$,

$x^2 = \frac{64}{4}$ $x^2 = 16$ $x = \pm 4$

Intercepts: $\{(0,-2), (0,2), (-4,0), (4,0)\}$

6. (2 points) List the intercepts for the graph of the equation $y = \frac{7x}{x^2 + 49}$.

$(0,0)$

7. (3 points) Determine whether the graph of $y^2 - x - 81 = 0$ is symmetric with respect to the x-axis, the y-axis, and/or the origin.

$y^2 = x + 81$ x axis symmetry

$y = \pm\sqrt{x+81}$

Since If $(\sqrt{x+81}, y)$ is on the graph

$(-\sqrt{x+81}, y)$ is on the graph

8. (4 points) The vertices of a triangle $\triangle ABC$ are $A(2, 1)$, $B(-2, 3)$ and $C(4, 5)$. Find the equation (in point-slope form) of the median through the vertex A. (Hint: the median goes to the midpoint of the opposite side)

$M\left(\frac{-2+4}{2}, \frac{3+5}{2}\right) = (1, 4)$

$\frac{\Delta y}{\Delta x} = \frac{4-1}{1-2} = \frac{3}{-1} = -3$

$y - 1 = -3(x-2)$ (or $y - 4 = -3(x-1)$)

9. (2 points) If a graph is symmetric with respect to the y-axis and it contains the point $(5, -6)$, find another point that must be on the graph.

$(-5, -6)$

10. (2 points) If $(3, b)$ is a point on the graph of $3x - 2y = 17$,what is b?

$3(3) - 2b = 17$

$b = \frac{17 - 9}{-2} = -4$

11. A vendor has learned that, by pricing hot dogs at \$1.25, sales will reach 124 hot dogs per day, but raising the price to \$2.00 will cause the sales to fall to 94 hot dogs per day. Let y be the number of hot dogs the vendor sells at x dollars each.

(a) (3 points) Write a linear equation that relates y, the number of hot dogs sold per day, to x, the price.

Using points $(1.25, 124)$ and $(2, 94)$

$\frac{\Delta y}{\Delta x} = \frac{124 - 94}{1.25 - 2} = -40$

$y = -40(x - 2) + 94$

or $y = -40x + 174$

(b) (2 points) Show how to use this equation to estimate how many hot dogs are sold if the price were \$1.50.

If $x = 1.5$

$y = -40(1.5) + 174 = 114$

About 114 hot dogs would be sold if the price was \$1.50

(c) (2 points) Show how to use this equation to find the best price to sell at least 75 hot dogs.

$y > 75$

$-40x + 174 > 75$

$-40x > -99$

$x < 2.475$

To sell at least 75 hot dogs, the price should be \$2.47 or less

12. (2 points) What is the point-slope form of the line that connects $(-3, 2)$ and $(1, -5)$?

$$y + 5 = -\frac{7}{4}(x - 1)$$

or $$y - 2 = -\frac{7}{4}(x + 3)$$

13. (2 points) What is the general form of the line that connects $(-3, 2)$ and $(1, -5)$?

$$7x + 4y = -13$$

14. (3 points) What is the general form of the equation of a circle that is centered at $(3, -7)$ with radius $2\sqrt{3}$?

$$(x-3)^2 + (y+7)^2 = 12$$

$$x^2 - 6x + 9 + y^2 + 14y + 49 = 12$$

$$x^2 + y^2 - 6x + 14y + 46 = 0$$

15. (3 points) What is the center and radius of the circle $x^2 + y^2 - 8x + 6y - 11 = 0$?

$(x - 8x + 16) + (y^2 + 6y + 9) = 16 + 9 + 11$

$(x-4)^2 + (y+3)^2 = 36$

Center $(4, -3)$

radius 6

16. (3 points) Consider $-7x + 2y = 42$

(a) What is the coordinate of the x intercept?

Let $y = 0$

$-7x = 42$

$x = -6$

$(-6, 0)$

(b) What is the coordinate of the y intercept?

Let $x = 0$

$2y = 42$

$y = 21$

$(0, 21)$

(c) What is the slope of this line?

$$\frac{\Delta y}{\Delta x} = \frac{21}{-6} = \frac{-7}{2}$$

17. (6 points) There are two lines that pass through the point $(6, -2)$, and whose sum of their intercepts is 5 (i.e, if the intercepts are $(0, a)$ and $(b, 0)$, then $a + b = 5$). Find the general form of these lines.

$a + b = 5$

$b = 5 - a$

$m = \frac{-b}{a} = \frac{+(5-a)}{a} = \frac{+2}{6-a}$

$(6-a)(5-a) = 2a$

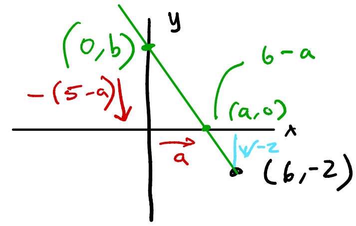
$30 - 6a - 5a + a^2 = 2a$

$a^2 - 13a + 30 = 0$

$(a-3)(a-10) = 0$

$a = 3$ or 10

$b = 2$ or -5

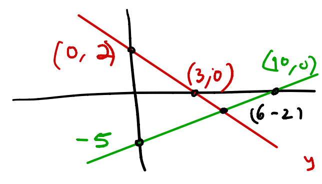


If slope is $-\frac{2}{3}$

$y + 2 = -\frac{2}{3}(x - 6)$

If slope is $\frac{5}{10} = \frac{1}{2}$

$y + 2 = \frac{1}{2}(x - 6)$



$y = \frac{1}{2}x - 5$

$y = -\frac{2}{3}x + 2$

18. (4 points) The amount of paint needed to cover the walls of a room varies jointly as the perimeter of the room and the height of the wall. If a room with a perimeter of 70 feet with 10-foot walls require 7 quarts of paint, find the amount of paint needed to cover the walls of a room with a perimeter of 65 feet with 6-foot walls.

P = paint in quarts, p = perimter in feet, h = height in feet

$P = kph$

$7 = k(70)(10)$

$k = \frac{7}{700} = \frac{1}{100}$

$P = \frac{1}{100}(65)(6)$

$P = \frac{390}{100} = 3.9$ quarts

It would take 3.9 quarts of paint to cover a permeter of 70 w/ 10 foot walls

19. (3 points) If the intercepts of a line are $\left(\frac{2}{3}, 0\right)$ and $\left(0, \frac{3}{4}\right)$, find the general equation of the line.

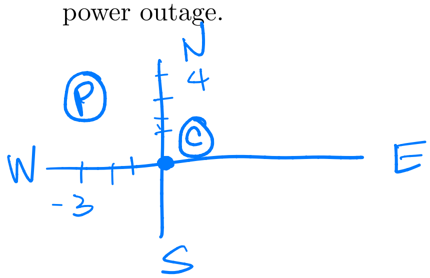
$$\frac{\Delta y}{\Delta x} = \frac{\frac{3}{4} - 0}{0 - \frac{2}{3}} = \frac{3}{4} \cdot \frac{-3}{2} = \frac{-9}{8}$$

$$y = -\frac{9}{8}\left(x - \frac{2}{3}\right)$$

$$8y = -9x + 6$$

$$9x + 8y = 6$$

20. (3 points) A power outage affected all homes and businesses within a 2 mile radius of the power station. If the power station is located 3 mile west and 4 miles north of the center of town (let the center of town be $(0, 0)$), find an equation of the circle consisting of the furthest points from the station affected by the power outage.



$$(x+3)^2 + (y-4)^2 = 4$$

21. (4 points) The amount of water used to take a shower is directly proportional to the amount of time that the shower is in use. A shower lasting 17 minutes requires 8.5 gallons of water. How much water is used in a shower lasting 5 minutes?

$$W = k\,t$$

$$8.5 = 17k$$

$$k = \frac{8.5}{17} = \frac{1}{2}$$

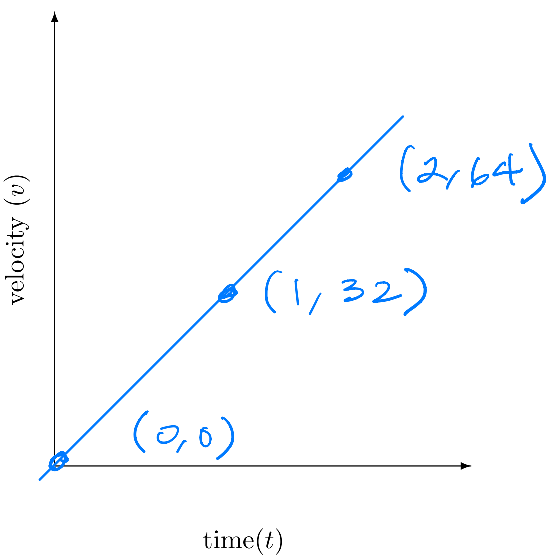
$$W = \frac{1}{2}(5) = \frac{5}{2} \text{ min}$$

22. (8 points) The velocity v of a falling object is directly proportional to the time t of the fall. After 2 seconds, the velocity of the object is 64 feet per second.

(a) Find an equation that relates the velocity v to the time t.

$$v(t) = kt$$
$$64 = 2k$$
$$k = 32.$$
$$v = 32t$$

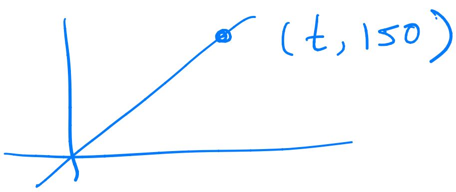
(b) Graph the equation and plot at least three points:



(c) Show how to use the equation to find the velocity v after 3 seconds.

$v = 32(3) = 96$ feet per seconds

(d) Using this equation, find how long something has been falling if the velocity is 150 feet per second.



$$150 = 32t$$
$$t = \frac{150}{32}$$
$$t = 4.6875 \text{ sec.}$$

23. (8 points) The rate of vibration of a string under constant tension varies inversely with the length of the string.

(a) Write an equation relating the frequency of vibration f with the length of the string l, using k as the constant of proportionality.

$$f(\ell) = \frac{k}{\ell}$$

(b) If a string is 36 inches long and vibrates 282 times per second, find k, the constant of proportionality.

$$282 = \frac{k}{36}$$

$$k = 10,152$$

(c) Find the length of a string that vibrates 440 times per second:

$$440 = \frac{10,152}{\ell} \quad , \quad \ell = \frac{1052}{440} = 23.0\overline{72}$$

inches

(d) Find the frequency of a string that is 18 inches long:

$$f(18) = \frac{10\,152}{18} = 562$$

times per second

 92.95 is more accurate

24. (6 points) Kepler's Third Law of Planetary Motion states that the square of the period of revolution T of a planet varies directly with the cube of its mean distance a from the Sun. If the mean distance of Earth from the Sun is 93 million miles (whose period is 365 days), what is the mean distance of the planet Venus from the Sun, given that Venus has a "year" (or period) of about 225 days? *(Hint: Use the same strategy as the last two problems– round your answer to the nearest hundred thousand miles)*

$$T^2 = k d^3$$

d = mean distance in Millon miles

T = period in days

$$\oplus: \quad (365)^2 = k(93)^3$$

$$k = \frac{365^2}{93^3} = 0.165629192 \text{ or } \frac{133225}{804357}$$

$$♀: \quad 225^2 = k\,d^3$$

$$d = \sqrt[3]{\frac{225^2}{k}} = 67.36113316 \text{ Million miles}$$

Kepler would estimate that the average distance to the sun is about 67,361,133 miles (or about 67 million miles)